The Nonlinear Mathematical Model of The Physical Body Under Variable Loading in Neutron Flux

Moh'd Isshaq Abu Halawa

***Abstract*— The influence of neutron irradiation upon (nonlinear physical) elastoplastic deformation of components of constructions under cyclic force disturbance is considered. As an example the problem of cyclic irradiation-force winding consider unsymmetrical with respect to thickness sandwich beam with external bearing layers made of metal and incompressible with respect to thickness internal layer (filler) made of polymer. Well effects resulting in appearance of additional volume deformation. Furthermore the differences used during the numerical solving reflects of the nature of the physical structure of body under influence the flux. On basis of experimental data the formula of irradiation reinforcement is suggested. The result may be extended in case of any given n-cyclic loading The theorem about variable loading is proved. Theorem about cyclic loadings of elastoplastic body in neutron flux allows to simplify essentially a whole class of boundary problems. Numerical solving for sandwich nonlinearly beam is adduced.**

***Index Terms*— irradiation-force, neutron flux, numerical solving, , variable loading.**

I. Introduction

Radiation treatment of rigid body is conducted by numerous effects resulting in appearance of additional volume deformation, changing of elastic and especially plastic properties of substance. Therefore it is necessary to bring in appropriate corrections into posing and solving of boundary-value problems concerning single and cyclic loading of elastoplastic components of constructions. The main factors are irradiation reinforcement of substance (increasing of yield point) and irradiation swelling (increasing of volume deformation). Below an attempt to extend the theory of variable loading by Moskvitin [1] upon discussed class of boundary-value problems.

II. Procedure of Methods of Solution.

A. Volume Deformation and Irradiation Reinforcement

Let's consider initially homogeneous isotropic body, occupying half-space $z \geq 0$. If upon the border ($z=0$) parallel to the axe z are felling neutrons with identical average energy and intention φ_0, the intention of neutrons, reaching the plain $z = const$, will be [2], [3] $\varphi(z)=\varphi_0 e^{-\mu z}$. The value μ is called macroscopic effective section and has valuation of 1/m.

If φ_0 is not dependent upon time, up to the instance t through the section z will pass the flow

$$I(z)=\varphi_0 t e^{-\mu z}. \tag{1}$$

In rough estimation we may consider, that changing of the volume of substance is directly proportional to the flux $I(z)$, and consequently $\theta_I = BI(z)$, where $B-$ is experimental constant. The value $I_0=\varphi_0 t$ will give total flux of neutrons upon unit of area of the surface of the body. In reactors φ_0 has the value of 10^{17}-10^{18} neutrons/(m^2 s), at that I_0 can reach the value of 10^{23}–10^{27} neutron /m^2, and θ_I approaching to value 0.1. Consequently, depending upon of energy of neutrons and properties of irradiated substance B can has the value of 10^{-28}–10^{-24} m^2/neutron.

Dependence of the modulus of elasticity, yield and solidity points and whole tension diagram upon I_0 for different energies was experimentally investigated after irradiating of samples in nuclear reactors. Results of experiments show that as usual the modulus of elasticity changes weakly (increase by 1.5-5 % relatively not irradiated sample). As for yield and solidity points – they are very sensitive to the irradiating and the yield point especially.

For massive bodies with flat boundary the number of neutrons passing at the depth z under this boundary is estimated by the formula (1), that's why the yield point will vary along z . At the surface of the body ($z=0$) the influence of irradiation upon the plastic limit σ_y is satisfactory characterised by the formula of irradiation reinforcement [2]:

$$\sigma_y = \sigma_{y0}\left[1+A\left(1-\exp(-\xi I_0)\right)^{1/2}\right]. \tag{2}$$

Here σ_{y0} - the plastic limit of not irradiated substance. At the depth z the formula takes the form

$$\sigma_y = \sigma_{y0}\left[1+A\left(1-\exp(-\xi I)\right)^{1/2}\right],$$

where the value of neutron flow $I(z)$ is characterised by formula (1). Let's denote appropriate values of deformation as $\varepsilon_{y0}, \varepsilon_s, A, \xi$ – are substance constants, taken from experiment. For example, if for aluminium alloy we accept $A=1.09$, $\xi = 9.73\times10^{-26}$ m^2/neutron, then the fig.1 indicates the satisfaction with known experimental data. Dark points – experimental data, solid line – the estimation by formula (2).

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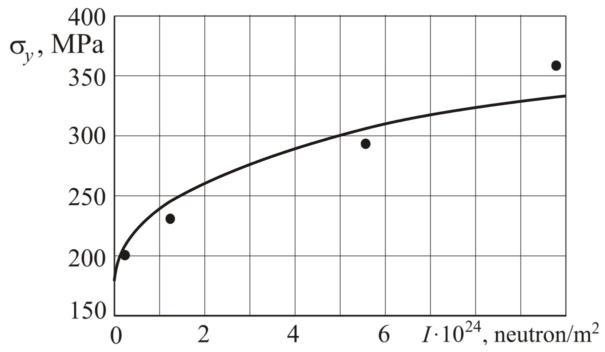

Fig. 1 experimental data, where dark points – experimental data, solid line – the estimation by formula (2)

B. Statement of problems of the theory of small-scale elastoplastic deformations

Let's consider the process of complex influence of external force and radiation loading upon deformable body within the theory of small-scale elastoplastic deformations. Suppose, that the body in natural state is influenced simultaneously by instant external forces F_i', R_i' with boundary moving u_{io}' and neutron flow with the value $I_0 = \varphi t$. It is supposed, that under such influence areas of elastic and plastic deformations appear in the body. Let's neglect the changing of modulus of elasticity caused by the neutron irradiation. Arising within the body stresses deformations and moving will be marked by one upper dash.

In elastic areas of solid body Hooke's law is valid and known equations, connecting deviators of stress and deformation s_{ij}', $э_{ij}'$ and also their spherical components σ', ε', are fulfilled $s_{ij}' = 2Gэ_{ij}'$, $\sigma' = K(3\varepsilon' - BI)$, with the correction due to additional volume deformation, caused by neutron irradiation BI. Here G denotes shear modulus, K – volumetric deformation modulus.

For those areas of solid body, where plastic deformations appeared the relation between deviators for simple loading can be represented as $s_{ij}' = 2Gэ_{ij}' f'(\varepsilon_u', I, a_k')$.

Here $f'(\varepsilon_u', I, a_k')$ – the plasticity function, depending upon deformation intensity ε_u', the value of neutron flow I and approximated parameters a_u'. In the conditions of simple (by A. A. Iluyshin) loading [1] this function will be universal, i. e. it can be find from experiments with extension, torsion etc.

So in the deformable body the relation between stresses and deformations under active loading from natural state and under influence of neutron flow in general case can be represented as

$$\begin{aligned} s_{ij}' &= 2Gэ_{ij}' f'(\varepsilon_u', I, a_k'), \\ \sigma' &= K(3\varepsilon' - BI) \end{aligned} \tag{3}$$

At that the plasticity function should be taken as $f'(\varepsilon_u', I, a_k') = 1$ in those areas, where $\varepsilon_u' \le \varepsilon_s'$, ε_s' – deformation, corresponding to the plastic limit at the start time.

If the force loading is rather quick (instantaneous) the irradiation reinforcement will not occur and originated areas of plastic deformations will be the same as in conditions without influence of neutron flow. Though if the active loading will be slow enough, external layers of the body will turn out to be reinforced and within these layers areas of plastic deformations will turn out to be smaller or will be missed at all, compared with the not irradiated body. There can take a place an effect when first plastic deformations will appear not on external reinforced surface, but under it, where the deformation intensity is great and plastic limit didn't increased.

So the influence of irradiation upon elastoplastic body is contrariwise to thermal, which decreases the plastic limit and results in increasing of areas of plastic deformations under equal loading.

Let's add to the relations in (3) differential equations and boundary conditions and also Cauchy proportions on the assumption of infinitesimal deformations

$$\begin{aligned} &\sigma_{ij,j}' + \rho F_i' = 0; \sigma_{ij}' l_j = R_i' \; on \, S_\sigma, \\ &u_i' = u_{0i}' \; on \, S_u; 2\varepsilon_{ij}' = u_{i,j}' + u_{j,i}' \end{aligned} \tag{4}$$

Comma in inferior index depicts the differentiation along next coordinate. Consider, that time variation of external loading and boundary moving occur in such a way, that appropriate loading trajectories are not related to the class of essentially complex loadings, and irradiation reinforcement takes place after force deformation of solid body. Hereinafter we shall suppose, that boundary problem (3) and (4) is solved.

C. Problem definition for repeated alternating-sign loading

Suppose that since $t = t_1$ the influence of neutron flow disappears ($\varphi = 0$), and external forces change so that in all points of plastically deformable areas of the body V_p' takes place the unloading and following alternating-sign loading by volumetric F_i'' and surface R_i'' forces (at S_σ) with boundary moving u_{oi}'' (at S_u). The level of irradiation of the body remains constant and is equal to the value before unloading $I_1 = \varphi t_1$. The plastic limit in points of the body depends upon coordinate z and becomes equal $\sigma_s''(I_1(z))$, i. e. it depends upon the value of deformation and irradiation reinforcement. The scheme of the process discussed is shown at the fig. 2.

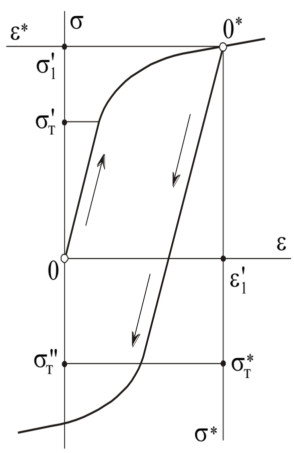

Fig. 2 The scheme of the process of the deformation and radiation hardening

Let's depict appropriate stresses, deformations and moving as $\sigma''_{ij}, \varepsilon''_{ij}, u''_i$. Formula (4) is valid for these values

$$\begin{aligned}&\sigma''_{ij,j} + \rho F''_i = 0; \sigma''_{ij} l_j = R''_i on S_\sigma,\\&u''_i = u''_{0i} \; on \, S_u; 2\varepsilon''_{ij} = u''_{i,j} + u''_{j,i}\end{aligned} \qquad (5)$$

Relation between stresses and deformations will be n curve of the second cycle.

$$s''_{ij} = 2G\text{э}''_{ij} \; f''(\varepsilon''_u, \varepsilon'_1, I_1, a''_k), \; \sigma'' = 3K\varepsilon''. \qquad (6)$$

Here $f''(\varepsilon''_u, \varepsilon'_1 I, a''_k)$ – plasticity function under repeated alternating-sign loading, depending upon the deformation intensity ε''_u , preceding values of deformation intensity ε'_1 and the level of irradiation of the body I_1, approximated parameters a''_k, describing the deformation.

At that plasticity function f' is supposed to be equal 1 in those areas, where new plastic deformations didn't appear, i. e. $\varepsilon''_u \le \varepsilon''_y$ modulo, ε''_y – deformation, corresponding to the plastic limit σ''_y under repeated loading.

Equations (5) and (6) define a boundary problem for values with two dashes. Complexity of the problem consists in the dependence of desired decision upon the unloading points ($\varepsilon'_1, \sigma'_1$), as the boundary problem must be defined and solved at every point the solid body. Let's discuss one of methods to avoid such difficulties.

For the values before the beginning of the unloading we shall retain designations $\sigma'_{ij}, \varepsilon'_{ij}, u'_i$. Following Moskvitin [1] we shall define the following differences for moment $t > t_1$:

$$s^*_{ij} = s'_{ij} - s''_{ij}, \text{э}^*_{ij} = \text{э}'_{ij} - \text{э}''_{ij} \qquad (7)$$

Let's depict physical state equations for values with asterisks. In zones V'_e and V''_e of unloading and elastic deformation are valid relations:

$$s^*_{ij} = 2G\text{э}^*_{ij}, \varepsilon^*_u \le \varepsilon^*_y(I_1),$$

in the area V''_p, where during variable loading the plastic deformation changes, must be valid relations:

$$s^*_{ij} = 2G\text{э}^*_{ij} \; f^*(\varepsilon^*_u, \varepsilon'_1, I_1, a^*_k). \qquad (8)$$

Here $f^*(\varepsilon^*_u, \varepsilon'_1, I_1, a^*_k)$, in general said, appears some new universal function, depicting nonlinearly of deformation diagram in axes $\sigma^* \sim \varepsilon^*$ (see the fig. 2). On the linear section $f^* = 0$.

In all points of the body the volumetric deformation remains elastic. Consequently before the beginning of unloading and for current state equalities are valid $\sigma' = K(3\varepsilon' - BI_1)$, $\sigma'' = K(3\varepsilon'' - BI)$, that's why for values with asterisks,

$$\sigma^* = 3K\varepsilon^*. \qquad (9)$$

Equilibrium equations, boundary conditions and Cauchy proportions for values $\sigma^*_{ij}, \varepsilon^*_{ij}, u^*_i$ will be

$$\begin{aligned}&\sigma^*_{ij,j} + \rho F^*_i = 0, \quad F^*_i = F'_i - F''_i\\&\sigma^*_{ij} l_i = R^*_i, \; R^*_i = R'_i - R''_i, on \; S_\sigma;\\&u^*_i = u^*_{0i} = u'_{0i} - u''_{0i}, on \; S_u; 2\varepsilon^*_{ij} = u^*_{i,j} + u^*_{j,i}.\end{aligned} \qquad (10)$$

Relations (8)–(10) make a new boundary problem for values with asterisks. If we now suppose that the function f^* in all points of deformation curve can be approximated by function f', that is to depict it by the same analytical equation but with another parameters a^*_k, we shall exclude the dependence of f^* upon ε_1:

$$f^* = f'(\varepsilon^*_u, I_1, a^*_k).$$

Comparing after this equation, the formulas (3) and (4) for the body with loading from natural state and equations for values with asterisks (8) – (10) we can mark, that they coincide with closeness within designations. That's why the solving of the problem for the values with asterisks can be get from known solving, of the problem, appropriated to loading from natural state by some replacements. For example, if the moving $u'_i = u'_i(x, \varepsilon'_u, \varepsilon'_s, I, a'_k)$ is known, then appropriate moving $u^*_i = u'_i(x, \varepsilon^*_u, \varepsilon^*_s, I_1, a^*_k)$ and under repeated alternating-sign loading is calculated from equation (7) $u''_i = u'_i - u^*_i$. Stresses and deformations are calculated by formulas of the same type.

III. RESULTS

Obtained result may be extended in case of any given *n*-cyclic loading (theorem about cyclic loadings of elastoplastic bodies in neutron flux). Suppose that under n-loading by external forces F^n_i, R^n_i with boundary moving u^n_{0i}, stresses σ^n_{ij}, deformations ε^n_{ij} and moving u^n_i appear. At the same time equilibrium equations, boundary conditions and Cauchy equations must be valid:

$$\begin{aligned}&\sigma^n_{ij,j} + \rho F^n_i = 0; \; \sigma^n_{ij} l_j = R^n_i \; on \, S_\sigma,\\&u^n_i = u^n_{0i} \; on \; S_u; \; 2\varepsilon^n_{ij} = u^n_{i,j} + u^n_{j,i}.\end{aligned} \qquad (11)$$

defined the following differences:

$$\sigma^{*n}_{ij} = (-1)^n(\sigma^{n-1}_{ij} - \sigma^n_{ij}), \quad \varepsilon^{*n}_{ij} = (-1)^n(\varepsilon^{n-1}_{ij} - \varepsilon^n_{ij}),$$

$$u^{*n}_i = (-1)^n(u^{n-1}_i - u^n_i).$$

Then the equations (11) turns out to be valid for the values with asterisks also:

$$\begin{aligned}
&\sigma_{ij}^{*n}{}_{,j} + \rho F_i^{*n} = 0,\\
&F_i^{*n} = (-1)^n (F_i^{n-1} - F_i^n);\\
&\sigma_{ij}^{*n} l_j = R_i^{*n} \; on\, S_\sigma, u_i^{*n} = u_{0i}^{*n} \; on\, S_u;\\
&R_i^{*n} = (-1)^n (R_i^{n-1} - R_i^n),\\
&u_{0i}^{*n} = (-1)^n (u_{0i}^{n-1} - u_{0i}^n);\\
&2\varepsilon_{ij}^{*n} = u_{i,j}^{*n} + u_{j,i}^{*n}.
\end{aligned} \quad (12)$$

Let's accept that under any n-loading the relation between sphere components of stresses and deformation tensors remains elastic. Repeating the previous supposition about possibility of curves $s'_{ij} \sim э'_{ij}$ and $s_{ij}^{*n} \sim э_{ij}^{*n}$ by nonlinearly functions of identical analytic type

$$s_{ij}^{*n} = 2G э_{ij}^{*n} f'\left(\varepsilon_u^{*n}, I_1, a_k^{*n}\right), \quad (13)$$

we shall conclude that the solving for the problem for values with asterisks (12) and (13) under any given n-loading may be get from the problem, concerning the loading from the natural state. For example, if the moving is known, $u'_i = u'_i(x, \varepsilon'_u, \varepsilon'_T, I, a'_k)$, then the appropriate value with asterisk will be $u_i^{*n} = u'_i(x, \varepsilon_u^{*n}, \varepsilon_T^{*n}, I_1, a_k^{*n})$.

After this the desired moving u_i^n can be calculated from equation

$$u_i^n = u'_i - \sum_{k=2}^{n} (-1)^k u_i^{*k} . \quad (14)$$

Stresses and deformations are calculated by formulas of the same type (14).

As an example the problem of cyclic irradiation-force winding of sandwich beam with one embed end (fig. 3). Unsymmetrical with respect to thickness sandwich beam with external bearing layers made of metal and incompressible with respect to thickness internal layer (filler) made of polymer is considered. For the description of the pack kinematics the hypothesis of broken normal line is accepted: in bearing layers Kirhgoff hypothesis is valid, in the filler normal line remains rectilinear without changing the length, but it turns on some additional angle $\psi(x)$. Bearing layers are accepted to be elastoplastic, the filler – elastic. Analytical solution of appropriate problem of theory of elasticity is depicted in [3]. Solving of the problem of small elastoplastic deformations under loading from natural state was found by method of elastic decisions. Aluminium alloy was used as a bearing layer, and Teflon was used as filler.

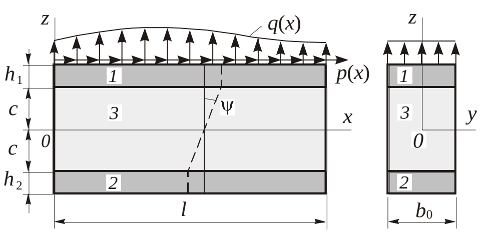

Fig. 3 cyclic irradiation-force winding of sandwich beam with one embed end

The solution of this problem for the theory of elasticity is known [4]-[6]:

$$\psi^{(n)}(x) = C_2^{(n)} sh(\beta x) + C_3^{(n)} + ch(\beta x) + $$
$$+ \frac{1}{\beta}\left[\begin{matrix} \mathrm{sh}(\beta x)\int g^{(n)} \mathrm{ch}(\beta x)dx - ch(\beta x \\ \int g^{(n)} sh(\beta x)dx \end{matrix}\right]$$

$$u^{(n)}(x) = \gamma_3 \psi^{(n)} +$$
$$+ \frac{1}{\alpha_2}\left[\begin{matrix} -a_4 L_2^{-1}(p - p_\omega^{(n-1)}) + \\ + a_7 L_3^{-1}(q - q_\omega^{(n-1)}) + \frac{a_7}{2} C_1^{(n)} x^2 \end{matrix}\right]$$
$$+ C_7^{(n)} x + C_8^{(n)} , \quad (15)$$

$$w^{(n)}(x) = \frac{1}{\alpha_2}[\alpha_1 \int \psi^{(n)} dx -$$
$$- a_7 L_3^{-1}(p - p_\omega^{(n-1)}) +$$
$$+ a_1 L_4^{-1}(q - q_\omega^{(n-1)}) + \frac{1}{6} a_1 C_1^{(n)} x^3] +$$
$$+ \frac{1}{2} C_4^{(n)} x^2 + C_5^{(n)} x + C_6^{(n)} x.$$

$$g^{(n)}(x) = \frac{\alpha_2}{\alpha_1}\gamma_1 h_\omega^{(n-1)} + \gamma_2 (p - p_\omega^{(n-1)}) +$$
$$\gamma_1 (\int (q - q_\omega^{(n-1)}) dx + C_1^{(n)}),$$

Here
$$N^\omega = \frac{4}{3} b_0 \sum_{k=1}^{3} G_k \int_{h_k} \omega_k \varepsilon_x^{(k)} dz ,$$
$$M^\omega = \frac{4}{3} b_0 \sum_{k=1}^{3} G_k \int_{h_k} \omega_k \varepsilon_x^{(k)} z dz , \quad (16)$$
$$Q^\omega = 2 b_0 G_3 \int_{-c}^{c} \omega_3 \psi dz .$$

Where n is approximation number; p_ω^{n-1} , h_ω^{n-1}, q_ω^{n-1} are called «additional» external stresses, assumed to be zero at the first step and are further calculated from the results of preceding approximation. For this aim, the formulas similar to (16) are used in which all summands have a superscript «$n-1$»:

$$p_\omega^{(n-1)} = \frac{1}{b_0} N,_x^{\omega(n-1)} ,$$

$$h_\omega^{(n-1)} = \frac{1}{b_0}(H,_x^{\omega(n-1)} - Q^{\omega(n-1)}) ,$$

$$q_\omega^{(n-1)} = \frac{1}{b_0} M,_{xx}^{\omega(n-1)} ,$$

$$N^{\omega(n-1)} = \sum_{k=1}^{3} N^{(k)(n-1)} =$$
$$= \tfrac{4}{3} b_0 \sum_{k=1}^{3} \int_{h_k} G_k \omega^{(k)} (\varepsilon_u^{(k)(n-1)}) \varepsilon_x^{(k)(n-1)} dz ,$$

$$M^{\omega(n-1)} = \sum_{k=1}^{3} M^{(k)(n-1)} =$$

$$= \frac{4}{3} b_0 \sum_{k=1}^{3} \int_{h_k} G_k \omega^{(k)} (\varepsilon_u^{(k)(n-1)}) \varepsilon_x^{(k)(n-1)} z dz \,,$$

$$H^{\omega(n-1)} = c(N^{(1)\omega(n-1)} - N^{(2)\omega(n-1)}) + + M^{(3)\omega(n-1)} \,,$$

$$Q^{\omega(n-1)} = 2b_0 \int_{-c}^{c} G_3 \omega^{(3)} (\varepsilon_u^{(3)(n-1)}) \psi^{(n-1)} dz \,,$$

Coefficients $\alpha_1, \alpha_2, \alpha_3, \beta^2, \gamma_1, \gamma_2, \gamma_3$ and linearly here integrates operators L_2^{-1}, L_3^{-1}, L_4^{-1} are determinates in [4]-[6].

Under the boundary conditions for edge-fixing of the plate, we obtain the following recurrent formulas for the integration constants $C_1^{(n)}, \ldots, C_8^{(n)}$:

$$
\begin{aligned}
C_1^{(n)} &= -L_1^{-1}(q - q_\omega^{(n-1)})\big|_{x=l} \,, \\
C_2^{(n)} &= \frac{1}{\beta}\left[\frac{\mathrm{ch}(\beta l)}{\mathrm{sh}(\beta l)}\left(\begin{array}{l}\int g^{(n)} \mathrm{sh}(\beta x) dx\big|_{x=l} \\ -\int g^{(n)} \mathrm{sh}(\beta x) dx\big|_{x=0}\end{array}\right) - \right. \\
&\left. -\int g^{(n)} ch(\beta x) dx\big|_{x=l}\right], \\
C_3^{(n)} &= \frac{1}{\beta}\int g^{(n)}(x) sh(\beta x) dx\big|_{x=0}, \\
C_4^{(n)} &= \frac{a_7}{\alpha_2} L_1^{-1}(p - p_\omega^{(n-1)})\big|_{x=l} - \\
&- \frac{a_1}{\alpha_2} L_2^{-1}(q - q_\omega^{(n-1)})\big|_{x=l} - \frac{a_1}{\alpha_2} C_1^{(n)} l, \\
C_5^{(n)} &= \frac{a_7}{\alpha_2} L_2^{-1}(p - p_\omega^{(n-1)})\big|_{x=0} - \\
&- \frac{a_1}{\alpha_2} L_3^{-1}(q - q_\omega^{(n-1)})\big|_{x=0}, \\
C_6^{(n)} &= \frac{a_7}{\alpha_2} L_3^{-1}(p - p_\omega^{(n-1)})\big|_{x=0} - \\
&- \frac{a_1}{\alpha_2} L_4^{-1}(q - q_\omega^{(n-1)})\big|_{x=0} - \frac{\alpha_1}{\alpha_2}\int \psi^{(n)} dx\big|_{x=0}, \\
C_7^{(n)} &= \frac{a_4}{\alpha_2} L_1^{-1}(p - p_\omega^{(n-1)})\big|_{x=l} - \\
&- \frac{a_7}{\alpha_2} L_2^{-1}(q - q_\omega^{(n-1)})\big|_{x=l} - \frac{a_7}{\alpha_2} C_1^{(n)} l, \\
C_8^{(n)} &= \frac{a_4}{\alpha_2} L_2^{-1}(p - p_\omega^{(n-1)})\big|_{x=0} - \\
&- \frac{a_7}{\alpha_2} L_3^{-1}(q - q_\omega^{(n-1)})\big|_{x=0}.
\end{aligned}
\qquad (17)
$$

Under the boundary conditions for edge-fixing of the plate, we obtain the following recurrent formulas for integration constants:

$$C_1^{(n)} = \frac{1}{l}\left(L_2^{-1}(q - q_\omega^{(n-1)})\big|_{x=0} - L_2^{-1}(q - q_\omega^{(n-1)})\big|_{x=l}\right),$$

$$C_4^{(n)} = \frac{a_7}{a_4}\left(D_1^{(n)}\big|_{x=0} - D_1^{(n)}\big|_{x=l}\right) - \frac{1}{a_4} L_2^{-1}(q - q_\omega^{(n-1)})\big|_{x=0} \,,$$

$$C_5^{(n)} = D_2^{(n)}\big|_{x=0} - D_2^{(n)}\big|_{x=l} - \frac{a_7 l}{2a_4}\left(D_1^{(n)}\big|_{x=0} - D_1^{(n)}\big|_{x=l}\right) - - \frac{l}{2a_4} L_2^{-1}(q - q_\omega^{(n-1)})\big|_{x=0} \,,$$

$$C_7^{(n)} = D_1^{(n)}\big|_{x=0} - D_1^{(n)}\big|_{x=l} \,,$$

$$D_1^{(n)} = \frac{a_7}{l\alpha_2} L_3^{-1}(q - q_\omega^{(n-1)}) - \frac{a_7 l}{2\alpha_2} L_2^{-1}(q - q_\omega^{(n-1)}) - - \frac{a_4}{l\alpha_2} L_2^{-1}(p - p_\omega^{(n-1)}) \,,$$

$$D_2^{(n)} = \frac{a_1}{l\alpha_2}\left(\int \psi^{(n)} dx + L_4^{-1}(q - q_\omega^{(n-1)})\right) - - \frac{a_1 l}{6\alpha_2} L_2^{-1}(q - q_\omega^{(n-1)}) - \frac{a_7}{l\alpha_2} L_3^{-1}(p - p_\omega^{(n-1)}) \,.$$

These constants remain form $C_2^{(n)}$, $C_3^{(n)}$, $C_6^{(n)}$, $C_8^{(n)}$ as in (17).

Appropriate mechanical properties of mediums are depicted in [2]. On figures (4) and (5) shear ψ and flexure w of sandwich beam, calculated with the help of different physical state equations, are shown. Curves with one dash corresponds to loading from natural state, with two dashes – repeated cyclic winding due to alternating-sign loading: *1'* – solving of elastic problem: *2'*– instant elastoplastic without irradiation; *3'*– elastoplastic winding of previously irradiated beam (I_1=5*10^{24} м$^{-2}$).

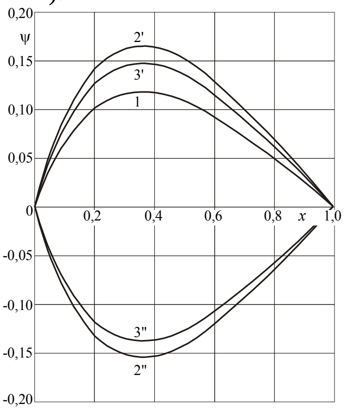

Fig. 4 shear ψ of sandwich beam

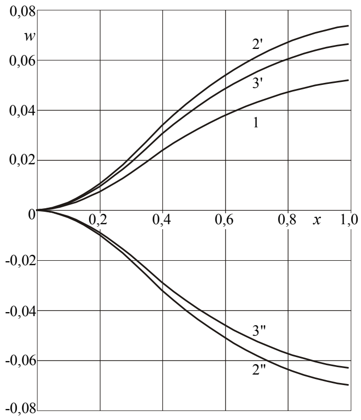

Fig. 5 flexure w of sandwich beam

Under combined influence of force loading and neutron flow during interval t_1 till the value I_1 deformation will correspond to the curve *2′*. Consequent instant unloading and force alternating-sign loading with level of irradiation I_1 will cause the shear and flexure of the beam, shown by curves *2*". If the beam under cyclic loading was irradiated beforehand, then deformation would correspond to the curve *3*".

IV. Conclusion

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